*Many ways to the top of the mountain, choose one....**Many ways to the top of the mountain, know many.*** Immanuel Williams

4.8 A Probability Notion:

Probability of "at least 1": This statement means one or more.

It is **EASIER** to find the complement of one or more.

Example 10: If a two parents had 3 children, what is the probability that at least 1 one the children is a girl (assuming boys and girls are equally likely).
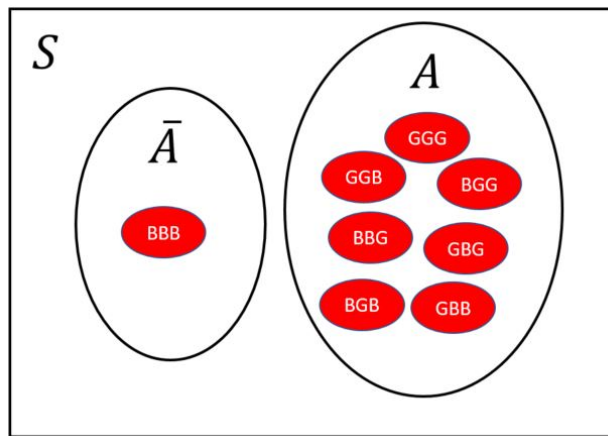
Event A: Getting at least 1 girl among 3 children.
Event $\bar{A}$: No girls among the 3 children. All children are Boys.

$$P(\bar{A}) = P(\text{Boy} \cap \text{Boy} \cap \text{Boy})$$
$$= P(\text{Boy})P(\text{Boy})P(\text{Boy}) \ (\underline{\hspace{2.5cm}})$$
$$= \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2}$$
$$= \underline{\hspace{3cm}}$$

P(Getting at least 1 girl among 3 children)= _____
$P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{8} = \frac{7}{8}$

Example 11: In a garden of 50 flowers, the probability that a flower does not bloom is $0.5\% = 0.005$. What is the probability of getting at least one flower that does not bloom in the garden of 50 flowers?

$P(1 \text{ flower not blooming}) = 0.005$
$P(1 \text{ flower blooming}) = 0.995$

Event A: At least 1 flower not blooming.
Event $\bar{A}$: All flowers blooming.

$$
\begin{aligned}
P(A) &= 1 - P(\bar{A}) \\
&= 1 - 0.995 \text{ x } 0.995 \text{ x } 0.995 \text{ x } 0.995... \text{ x } 0.995 \\
&= 1 - 0.995^{50} \\
&= 1 - 0.7783 \\
&= \underline{\hspace{5cm}}
\end{aligned}
$$

4.9   Conditional Probability & Bayes Theorem

Recall **Conditional Probability**:

- $P(A \text{ given } B) = P(A|B)$
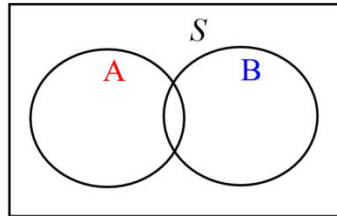- $P(B \text{ given } A) = P(B|A)$

These statements are equivalent:

$$
P(A \cap B) = P(A|B)P(B) \Leftrightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}
$$

$$
P(A \cap B) = P(B|A)P(A) \Leftrightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}
$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

$$P(B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$$

Bayes Theorem:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{8}$$

$$= \frac{P(B|A)P(A)}{P(A \cap B) + P(\bar{A} \cap B)} \tag{9}$$

$$= \frac{P(B|A)P(A)}{P(A)P(B|A) + P(\bar{A})P(B|\bar{A})} \tag{10}$$

$$\tag{11}$$

4.10   Risk & Odds

Imagine a prospective study.

	Disease	No Disease	
Treatment	a=100	b=400	a+b=100+400
Placebo	c=250	d=250	c+d=250+250

Risk: is given as a probability value.

$$p_t = P(\text{Disease}|\text{treatment}) = \frac{100}{100+400} = \frac{1}{5}$$

p_t : Proportion (or incidence rate) of some characteristic in the treatment group.
Example: Proportion of heart attacks in the treatment group

$$p_c = P(\text{Disease}|\text{control}) = \frac{250}{250+250} = \frac{1}{2}$$

p_c : Proportion (or incidence rate) of some characteristic in the control group.
Example: Proportion of heart attacks in the control group

Absolute Risk Reduction:

$$|P(\text{event in treatment}) - P(\text{event in control})| = |\frac{100}{100+400} - \frac{250}{250+250}|$$
$$= |p_t - p_c|$$

Relative Risk:

$$\text{Relative Risk} = \frac{p_t}{p_c} = \frac{\frac{1}{5}}{\frac{1}{2}} \text{ Rick Ratio}$$

Odds Ratio:

$$\text{Odds of Event} A = \frac{P(A)}{P(\bar{A})}$$

$$\text{Odds against Event} A = \frac{P(\bar{A})}{P(A)}$$

Relative Odds:

$$\text{Odds Ratio} = \frac{\text{odds in favor of treatment group}}{\text{odds in favor of control group}}$$

$$\text{Odds Ratio} = \frac{ad}{bc}$$

Use relative risk and/or odds ratio in Prospective studies.

Use odds ratio only in Retrospective studies. In this case the incidence rate of an event might be the result of the study design and not the true incidence rate.